

EXAM III
CALCULUS AB
SECTION I PART A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

- (1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1}x = \arcsin x$).

-
1. Which of the following is a function with a vertical tangent at $x = 0$?

(A) $f(x) = x^3$ (B) $f(x) = \sqrt[3]{x}$ (C) $f(x) = \frac{1}{x}$ (D) $f(x) = \sin x$ (E) $f(x) = \tan x$

Ans

☐

2. $\int_0^5 \frac{dx}{\sqrt{1+3x}} =$

(A) 4 (B) $\frac{8}{3}$ (C) 2 (D) $\frac{16}{9}$ (E) 1

Ans

☐

3. Which function is NOT continuous everywhere?

(A) $y = |x|$

(B) $y = x^{2/3}$

(C) $y = \sqrt{x^2 + 1}$

(D) $y = \frac{x}{x^2 + 1}$

(E) $y = \frac{4x}{(x+1)^2}$

Ans

☐

4. The area of the region bounded by the curve $y = e^{-x}$, the x -axis, the y -axis and the line $x = 2$ is equal to

(A) 1

(B) 2

(C) $\ln e^x$

(D) $\frac{1}{e^2} - 1$

(E) $1 - \frac{1}{e^2}$

Ans

☐

5. If $g(x) = x + \cos x$, then $\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} =$

(A) $\sin x + \cos x$

(B) $\sin x - \cos x$

(C) $1 - \sin x$

(D) $1 - \cos x$

(E) $x^2 - \sin x$

Ans

☐

6. $\int_0^4 \frac{2x}{x^2 + 9} dx =$

- (A) 25
- (B) 16
- (C) $\ln \frac{25}{9}$
- (D) $\ln 4$
- (E) $\ln \frac{5}{3}$

Ans

7. A function g is defined for all real numbers and has the following property:

$$g(a + b) - g(a) = 4ab + 2b^2. \text{ Find } g'(x).$$

- (A) 4
- (B) -4
- (C) $2x^2$
- (D) $4x$
- (E) does not exist

Ans

8. Given the function defined by $f(x) = x^5 - 5x^4 + 3$, find all values of x for which the graph of f is concave up.

- (A) $x > 0$
- (B) $x > 3$
- (C) $0 < x < 3$
- (D) $x < 0$ or $x > 3$
- (E) $x < 0$ or $x > 5$

Ans

9. If $f(x) = 2 + |x|$, find the average value of the function f on the interval $-1 \leq x \leq 3$.

(A) $\frac{7}{4}$ (B) $\frac{9}{4}$ (C) $\frac{11}{4}$ (D) $\frac{13}{4}$ (E) $\frac{15}{4}$

Ans

☐

10. A particle starts at $(5, 0)$ when $t = 0$ and moves along the x -axis in such a way that at time $t > 0$ its velocity is given by $v(t) = \frac{1}{1+t}$. Determine the position of the particle at $t = 3$.

(A) $\frac{97}{16}$
(B) $\frac{95}{16}$
(C) $\frac{79}{16}$
(D) $1 + \ln 4$
(E) $5 + \ln 4$

Ans

☐

11. If $g(x) = \sqrt[3]{x-1}$ and f is the inverse function of g , then $f'(x) =$

(A) $3x^2$
(B) $3(x-1)^2$
(C) $-\frac{1}{3}(x-1)^{-4/3}$
(D) $\frac{1}{3}(x-1)^{2/3}$
(E) does not exist

Ans

☐

12. Suppose $F(x) = \int_0^{\cos x} \sqrt{1+t^3} dt$ for all real x , then $F'\left(\frac{\pi}{2}\right) =$

- (A) -1
(B) 0
(C) $\frac{1}{2}$
(D) 1
(E) $\frac{\sqrt{3}}{2}$

Ans

☐

13. If the line $3x - y + 2 = 0$ is tangent in the first quadrant to the curve $y = x^3 + k$, then $k =$

- (A) 5
(B) -5
(C) 4
(D) 1
(E) -1

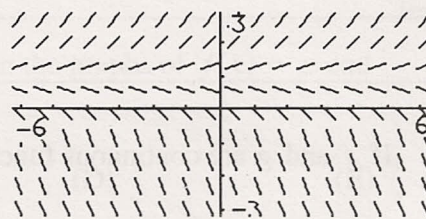
Ans

☐

14. The slope field for a differential equation $\frac{dy}{dx} = f(x, y)$ is given in the figure. Which of the following statements are true?

- I. A solution curve that contains the point $(0, 2)$ also contains the point $(-2, 0)$.
II. As y approaches 1 , the rate of change of y approaches zero.
III. All solution curves for the differential equation have the same slope for a given value of y .

- (A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III



Ans

☐

15. $\frac{d}{dx} [\text{Arctan } 3x] =$

(A) $\frac{1}{1+9x^2}$

(B) $\frac{3}{1+9x^2}$

(C) $\frac{3}{\sqrt{4x^2-1}}$

(D) $\frac{3}{1+3x}$

(E) none of the above

Ans

☐

16. $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 1} =$

(A) -2

(B) -1

(C) 10

(D) 1

(E) 2

Ans

☐

17. If f and g are continuous functions such that $g'(x) = f(x)$ for all x , then $\int_2^3 f(x) dx =$

(A) $g'(2) - g'(3)$

(B) $g'(3) - g'(2)$

(C) $g(3) - g(2)$

(D) $f(3) - f(2)$

(E) $f'(3) - f'(2)$

Ans

☐

18. Let $y = 2e^{\cos x}$. Both x and y vary with time in such a way that y increases at the constant rate of 5 units per second. The rate at which x is changing when $x = \frac{\pi}{2}$ is

(A) 10 units/sec
(B) -10 units/sec
(C) -2.5 units/sec
(D) 2.5 units/sec
(E) -0.4 units/sec

Ans

19. $\int_1^2 \frac{dx}{x^3} =$

(A) $\frac{3}{8}$
(B) $-\frac{3}{8}$
(C) $\frac{15}{64}$
(D) $\frac{3}{4}$
(E) $\frac{15}{16}$

Ans

20. The maximum distance, measured horizontally, between the graphs of $f(x) = x$ and $g(x) = x^2$ for $0 \leq x \leq 1$, is

(A) 1 (B) $\frac{3}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$ (E) $\frac{1}{8}$

Ans

21. Let f be the function defined by $f(x) = \begin{cases} x + 1 & \text{for } x < 0 \\ 1 + \sin \pi x & \text{for } x \geq 0. \end{cases}$ Then $\int_{-1}^1 f(x) dx =$

(A) $\frac{3}{2}$

(B) $\frac{3}{2} - \frac{2}{\pi}$

(C) $\frac{1}{2} - \frac{2}{\pi}$

(D) $\frac{3}{2} + \frac{2}{\pi}$

(E) $\frac{1}{2} + \frac{2}{\pi}$

Ans

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22. If $x^2 + 2xy - 3y = 3$, then the value of $\frac{dy}{dx}$ at $x = 2$ is

(A) 1

(B) 2

(C) -2

(D) $\frac{10}{3}$

(E) $-\frac{1}{2}$

Ans

☐

23. Let f be the function defined by $f(x) = x^{2/3}(5 - 2x)$. f is increasing on the interval

(A) $x < -\frac{5}{2}$ (B) $x > 0$ (C) $x < 1$ (D) $0 < x < \frac{5}{8}$ (E) $0 < x < 1$

Ans

☐

24. Let R be the region in the first quadrant bounded by the x -axis and the curve $y = 2x - x^2$. The volume produced when R is revolved about the x -axis is

(A) $\frac{16\pi}{15}$ (B) $\frac{8\pi}{3}$ (C) $\frac{4\pi}{3}$ (D) 16π (E) 8π

Ans

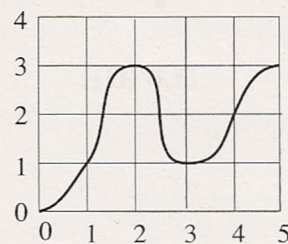
25. What are all values of k for which the graph of $y = 2x^3 + 3x^2 + k$ will have three distinct x -intercepts?

(A) all $k < 0$
(B) all $k > -1$
(C) all k
(D) $-1 < k < 0$
(E) $0 < k < 1$

Ans

26. Use the Trapezoid Rule with $n = 4$ to approximate the integral $\int_1^5 f(x) dx$ for the function f whose graph is shown at the right.

(A) 7
(B) 8
(C) 9
(D) 10
(E) 11

graph of $y = f(x)$ 

Ans

27. A point moves so that x , its distance from the origin at time t , $t \geq 0$ is given by:
 $x(t) = \cos^3 t$. The first time interval in which the point is moving to the right is

- (A) $0 < t < \frac{\pi}{2}$
(B) $\frac{\pi}{2} < t < \pi$
(C) $\pi < t < \frac{3\pi}{2}$
(D) $\frac{3\pi}{2} < t < 2\pi$
(E) none of these

Ans

☐

28. The function f is defined by $f'(x) = (x - 2)^2(x - 7)^3$. The graph of f has an inflection point where $x =$

- (A) 4 only
(B) 7 only
(C) 2 and 4 only
(D) 2 and 7 only
(E) 2 and 4 and 7

Ans

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EXAM III
CALCULUS AB
SECTION I PART B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
- (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
- (3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix “arc” (e.g., $\sin^{-1}x = \arcsin x$).

-
1. The derivative of the function g is $g'(x) = \cos(\sin x)$. At the point where $x = 0$ the graph of g
- I. is increasing, II. is concave down, III. attains a relative maximum point.
- (A) I only (B) II only (C) III only (D) I and III only (E) I, II, III

Ans

-
2. The approximate *average* rate of change of the function $f(x) = \int_0^x \sin(t^2) dt$ over the interval $[1, 3]$ is
- (A) 0.19 (B) 0.23 (C) 0.27 (D) 0.31 (E) 0.35

Ans

3. When $\int_{-1}^5 \sqrt{x^3 - x + 1} \, dx$ is approximated by using the mid-points of 3 rectangles of equal width, then the approximation is nearest to

(A) 22.6 (B) 22.9 (C) 23.2 (D) 23.5 (E) 23.8

Ans

☐

4. Find the total area between the graph of the curve $y = x^3 - 5x^2 + 4x$ and the x -axis.

(A) 11.74
(B) 11.77
(C) 11.80
(D) 11.83
(E) 11.86

Ans

☐

5. The graph of $y = \frac{\sin x}{x}$ has

I. a vertical asymptote at $x = 0$
II. a horizontal asymptote at $y = 0$
III. an infinite number of zeros

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Ans

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6. The graph of the function f is shown at the right. The graphs of the five functions:

$$y = f(x+1),$$

$$y = f(x) + 1,$$

$$y = f(-x),$$

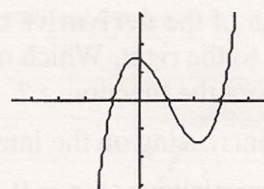
$$y = f'(x) \text{ and}$$

$$y = \int_1^x f(t) dt$$

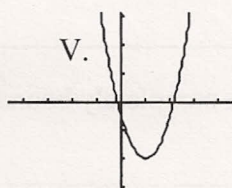
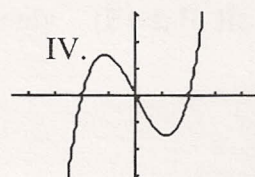
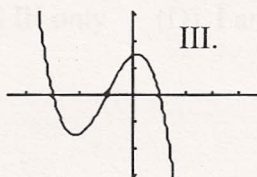
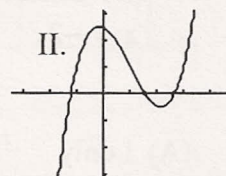
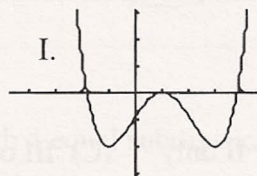
are shown in the *wrong* order.

The correct order is

- (A) II, IV, III, V, I
 (B) IV, II, III, I, V
 (C) IV, II, III, V, I
 (D) IV, III, II, V, I
 (E) II, IV, III, I, V



the graph of f



Ans

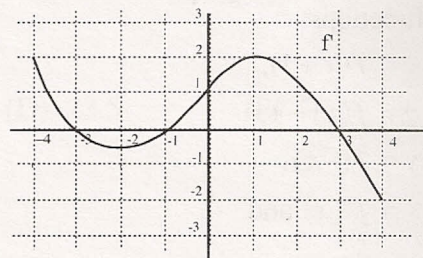
7. The region in the first quadrant bounded above by the graph of $y = \sqrt{x}$ and below by the x -axis on the interval $[0, 4]$ is revolved about the x -axis. If a plane perpendicular to the x -axis at the point where $x = k$ divides the solid into parts of equal volume, then $k =$

- (A) 2.77 (B) 2.80 (C) 2.83 (D) 2.86 (E) 2.89

Ans

8. The graph of the **derivative** of a function f is shown to the right. Which of the following is true about the function f ?

- I. f is increasing on the interval $(-2, 1)$.
 II. f is continuous at $x = 0$.
 III. The graph of f has an inflection point at $x = -2$.

graph of the derivative of f 

- (A) I only (B) II only (C) III only (D) II and III only (E) I, II, III

Ans

☐

9. The area of the region completely bounded by the curve $y = -x^2 + 2x + 4$ and the line $y = 1$ is

- (A) 8.7
 (B) 9.7
 (C) 10.7
 (D) 11.7
 (E) 12.7

Ans

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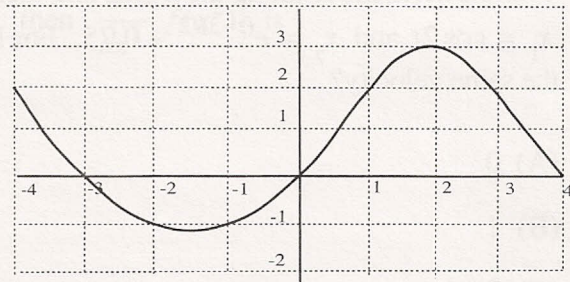
10. If functions f and g are defined so that $f'(x) = g'(x)$ for all real numbers x with $f(1) = 2$ and $g(1) = 3$, then the graph of f and the graph of g

- (A) intersect exactly once;
 (B) intersect no more than once;
 (C) do not intersect;
 (D) could intersect more than once;
 (E) have a common tangent at each point of tangency.

Ans

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11. The graph of a function f whose domain is the interval $[-4, 4]$ is shown in the figure. Which of the following statements are true?

the graph of the function f 

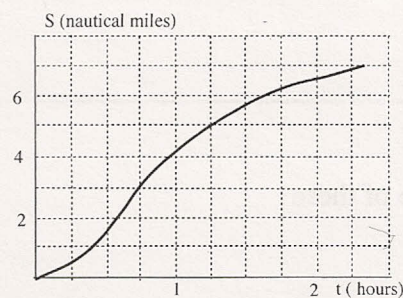
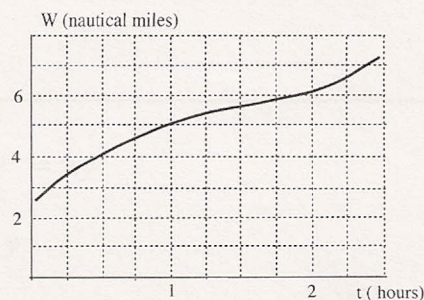
- I. The average rate of change of f over the interval from $x = -2$ to $x = 3$ is $\frac{1}{5}$.
- II. The slope of the tangent line at the point where $x = 2$ is 0.
- III. The left-sum approximation of $\int_{-1}^3 f(t) dt$ with 4 equal subdivisions is 4.

(A) I only (B) I and II only (C) II and III only (D) I and III only (E) I, II, III

Ans

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12. One ship traveling west is $W(t)$ nautical miles west of a lighthouse and a second ship traveling south is $S(t)$ nautical miles south of the lighthouse at time t (hours). The graphs of W and S are shown below. At what approximate rate is the distance between the ships increasing at $t = 1$? (nautical miles per hour = knots)



(A) 1 knot (B) 4 knots (C) 7 knots (D) 10 knots (E) 13 knots

Ans

☐

13. Two particles move along the x -axis and their positions at time $0 \leq t \leq 2\pi$ are given by $x_1 = \cos 2t$ and $x_2 = e^{(t-3)/2} - 0.75$. For how many values of t do the two particles have the same velocity?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Ans

14. The line $x - 2y + 9 = 0$ is tangent to the graph of $y = f(x)$ at $(3, 6)$ and is also parallel to the line through $(1, f(1))$ and $(5, f(5))$. If f is differentiable on the closed interval $[1, 5]$ and $f(1) = 2$, find $f(5)$.

(A) 2
(B) 3
(C) 4
(D) 5
(E) none of these

Ans

15. If $\frac{d}{dx}[f(x)] = g(x)$ and $\frac{d}{dx}[g(x)] = f(3x)$, then $\frac{d^2}{dx^2}[f(x^2)]$ is

(A) $4x^2 f(3x^2) + 2g(x^2)$

(B) $f(3x^2)$

(C) $f(x^4)$

(D) $2x f(3x^2) + 2g(x^2)$

(E) $2x f(3x^2)$

Ans

☐

16. The point $(1, 9)$ lies on the graph of an equation $y = f(x)$ for which $\frac{dy}{dx} = 4x\sqrt{y}$ where $x \geq 0$ and $y \geq 0$. When $x = 0$ the value of y is

(A) 6

(B) 4

(C) 2

(D) $\sqrt{2}$

(E) 0

Ans

☐

17. Find $\frac{dy}{dx}$ for $e^y = xy$.

(A) $\ln x + \ln y$

(B) $\frac{x+y}{xy}$

(C) $\frac{xy}{x+y}$

(D) $\frac{xy-x}{y}$

(E) $\frac{y}{xy-x}$

Ans

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EXAM III
CALCULUS AB
SECTION II, PART A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

- Before you begin Part A of Section II, you may wish to look over the problems before starting to work on them. It is not expected that everyone will be able to complete all parts of all problems and you will be able to come back to Part A (without a calculator), if you have time after Part B. All problems are given equal weight, but the parts of a particular solution are not necessarily given equal weight.
- You should write all work for each problem in the space provided. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed out work will not be graded.
- SHOW ALL YOUR WORK. Clearly label any functions, graphs, tables, or other objects you use. You will be graded on the correctness and completeness of your methods as well as your final answers. Answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons.
- You are permitted to use your calculator in Part A to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate in your exam booklet the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example,
$$\int_1^5 x^2 dx$$
 may not be written as $\text{fnInt}(X^2, X, 1, 5)$.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified.
- If you use decimal approximations in your calculations, you will be graded on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

THE EXAM BEGINS ON THE NEXT PAGE

PLEASE TURN OVER

-
1. Let R be the region in the first quadrant bounded above by the graph of $f(x) = 3 \cos x$ and below by the graph of $g(x) = e^{x^2}$.
- (a) Find the area of region R .
 - (b) Set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x -axis.
 - (c) Let the base of a solid be the region R . If all cross sections perpendicular to the x -axis are squares, set up, but do not integrate an integral expression in terms of a single variable for the volume of the solid.
-

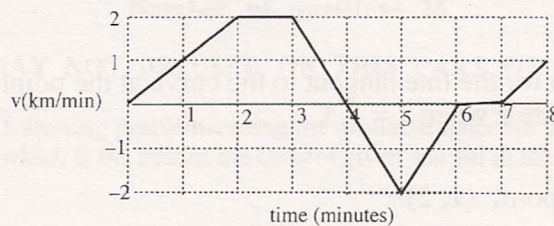
2. Let f be the function defined by $f(x) = \ln(x+1) - \sin^2 x$ for $0 \leq x \leq 3$.
- (a) Find the x -intercepts of the graph of f .
 - (b) Find the intervals on which f is increasing.
 - (c) Find the absolute maximum and the absolute minimum value of f . Justify your answer.
-

-
3. Let $f(x) = 4 - x^2$. For $0 < p < 2$, let $A(p)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(p, 4 - p^2)$.
- (a) Find $A(2)$.
- (b) For what value of p is $A(p)$ a minimum?
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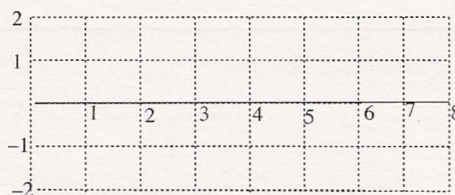
A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK
ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

4. Let $f(x) = x^3 + px^2 + qx$.
- (a) Find the values of p and q so that $f(-1) = -8$ and $f'(-1) = 12$.
 - (b) Find the value of p so that the graph of f changes concavity at $x = 2$.
 - (c) Under what conditions on p and q will the graph of f be increasing everywhere.
-

5. A car is moving along a straight road from A to B , starting from A at time $t = 0$. Below is a graph of the car's velocity (positive direction from A to B), plotted against time.



- (a) How many kilometers away from A is the car at time $t = 6$?
(b) At what time does the car change direction? Explain briefly.
(c) On the axes provided, sketch a graph of the acceleration of the car.



-
6. Consider the curve given by the equation $y^3 - 3xy = 2$.
- (a) Find $\frac{dy}{dx}$.
 - (b) Write an equation for the line tangent to the curve at the point $(1, 2)$. Use this tangent line to approximate y when $x = 1.3$.
 - (c) Find $\frac{d^2y}{dx^2}$ at the point $(1, 2)$.
 - (d) Is your approximation in Part (b) an overestimate or an underestimate. Justify your answer.
-